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Shubnikov–de Haas Frequency Anisotropy in GaSb

D. G. SEILER
Department of Physics, North Texas State University, Denton, Texas 76203
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Measurements, which are more precise than those previously reported, of the Shubnikov–de Haas frequency anisotropy in n-type GaSb are presented. The observed anisotropy is found to be in excellent agreement with a warped model of the Fermi surface recently presented by Seiler and Becker.

I. INTRODUCTION

IN a recent paper by Seiler and Becker (hereafter referred to as SB)[1] it was shown that the Shubnikov–de Haas (SdH) periods or frequencies of samples of n-type tellurium-doped GaSb depends on the magnetic field direction. With the field along most directions, the calculations based on a warping model and the experimental frequency data were found to be in good agreement (see Figs. 4–6 in SB). However, there was some scatter in their experimental values of the frequency anisotropy. When the magnetic field was along or close to the ⟨111⟩ and ⟨110⟩ crystallographic directions, the experimental frequency anisotropy did not satisfactorily agree with the warping model they used. The experimental data were relatively flat in this region, with no pronounced anisotropy observable (see Figs. 4 and 5 in SB).

More recently, Zhang[2] made a direct energy-band calculation for GaSb and calculated SdH periods for the field along ⟨001⟩, ⟨110⟩, and ⟨111⟩ directions for two of the samples used, 24B and 23B. His calculated values agreed very closely (within 0.05%) with the experimental periods reported in SB.

The present study was undertaken to obtain more accurate values of the frequency anisotropy than were obtained in SB. The results obtained from the study are then directly compared to the previous warping model given in SB and also to Zhang's theoretically calculated values of SdH frequencies.

II. FREQUENCY ANISOTROPY MEASUREMENT

A. Comments on Previous Measurements Used by SB

The periods or frequencies reported in SB were calculated by measuring the phase change of the oscillations between two known magnetic fields. These periods were calculated the same way in all field directions, except for angular positions close to and parallel to the ⟨111⟩ magnetic field directions. For $\mathbf{B}\|\langle 111\rangle$ directions, it was shown that an SdH amplitude minimum occurs for the high-concentration n-type GaSb used in SB.[3,4] For example, for a sample with $R_{4.2°K}=-3.2$ cm^3/C, a minimum occurred at 11 kG and for a sample with $R_{4.2°K}=-5.1$ cm^3/C, a minimum occurred at 8.8 kG. None occurred at higher fields. Thus, analysis of the period data for this orientation had to be confined to fields well above 11 or 8.8 kG to avoid the harmonic distortion (see, for example, Fig. 1, Ref. 3) in the region of the amplitude minimum. Consequently, the data reported for $\mathbf{B}\|\langle 111\rangle$ were not calculated over the same range of magnetic fields as for the other field directions. The distortion present near the SdH amplitude minima might also have slightly influenced the values of the periods or frequencies for $\mathbf{B}\|\langle 111\rangle$. Thus, a more precise way of measuring the frequency anisotropy than that used in SB is needed to examine the detailed nature of the SdH frequency anisotropy.

B. Accurate Frequency Anisotropy Measurement

The oscillatory behavior is given by $A_0(B)\sin(2\pi F/B+\phi_0)$, where $A_0(B)$ is the observed oscillatory amplitude (which includes the influence of the Bessel-function modulation caused by the field modulation and phase sensitive detection), F is the SdH frequency, and ϕ_0 is the phase angle (theoretically it is $-\frac{1}{4}\pi$). Some data were presented in SB (Fig. 8) to show that ϕ_0 can be assumed to be constant as a function of magnetic field direction. Then, for the oscillatory behavior at a nodal position for the field along a direction B_1,

$$\sin(2\pi F_1/B_1+\phi_0)=0$$

or

$$2\pi F_1/B_1+\phi_0=N\pi\,, \tag{1}$$

where N is an integer. If the magnetic field direction is changed to lie along a direction B_2, F_1 changes to F_2. For the same constant-phase nodal position (i.e., the same value of N), we have

$$2\pi F_2/B_2+\phi_0=N\pi\,. \tag{2}$$

Subtracting Eq. (2) from Eq. (1) and rearranging gives

$$F_1/F_2=B_1/B_2\,. \tag{3}$$

Thus, if $\mathbf{B}_2\|\langle 001\rangle$ and B_1 is at an angle θ from an ⟨001⟩ axis, then

$$F(\theta)/F(001)=B(\theta)/B(001)\,. \tag{4}$$

Thus, a simple measurement of the magnetic field strengths as a function of θ at a constant phase point of the oscillations will give directly the frequency ratio

[1] D. G. Seiler and W. M. Becker, Phys. Rev. **183**, 784 (1969).
[2] H. I. Zhang, Phys. Rev. (to be published).
[3] D. G. Seiler and W. M. Becker, Phys. Letters **26A**, 96 (1967).
[4] D. G. Seiler, W. M. Becker, and L. M. Roth, Phys. Rev. (to be published).

of interest. This is in contrast with the previous calculation of $F(\theta)/F(001)$ in SB. There the individual frequencies were first calculated and then the frequency ratio formed by dividing one frequency into another.

III. RESULTS AND DISCUSSION

The experimental recordings of the SdH oscillations that were taken previously by SB were used to find $B(\theta)/B(001)$ for a constant phase point. This phase point was in the vicinity of 19 kG. The ratio $B(\theta)/B(001)$ can be found quite accurately since magnetic field strengths were measured with NMR techniques. The values of $B(\theta)/B(001)$ (represented by circles) are shown in Fig. 1. To check the reproducibility of the data recording and calculations of $B(\theta)/B(001)$, two field sweeps were made at one particular angle for each of the three samples. For sample 24B, at $\theta=90°$, the two resulting values of $B(\theta)/B(001)$ for the two sweeps were 1.0088 and 1.0087; for 23B, at $\theta=90°$, they were 1.0076 and 1.0075; for 80B, at $\theta=199°$, they were 1.0024 and 1.0024.

The warping model employed in SB led to a SdH frequency ratio that could be expressed as

$$\frac{F(\theta)}{F(001)}=\frac{[1-C_1g_1(\theta)/2\pi]}{[1-\frac{1}{8}C_1]}. \tag{5}$$

Here C_0 and C_1 only depend on the Fermi energy and band parameters and $g_1(\theta)$ (θ defines the magnetic field direction) gives the angular dependence caused by the warping term in the energy expression. We define a quantity X as

$$X=\frac{F(111)-F(001)}{F(001)}=\frac{F(111)}{F(001)}-1. \tag{6}$$

Since $g_1(001)=\frac{1}{4}\pi$ and $g_1(111)=\frac{1}{2}\pi$, Eq. (5) and Eq. (6) can be solved for C_1 giving

$$C_1=-8X/(1-X). \tag{7}$$

From the experimental data shown in Fig. 1, a value of X and hence a value of C_1 were obtained. Substituting this value of C_1 into Eq. (5) then gives the theoretical angular variation of $F(\theta)/F(001)$, which is represented by the curved line in Fig. 1. The new experimental values of C_1 are not too different from those reported earlier (see Table II in SB), as seen in Table I. Figure 1 also shows the theoretical frequency ratio results which Zhang obtained from his energy-band calculations.[5]

TABLE I. Comparison of C_1 values obtained previously in Ref. 1 with those reported in this study.

Sample	C_1 (old)	C_1 (new)
24B	−0.086	−0.090
23B	−0.072	−0.078
80B	−0.081	−0.079

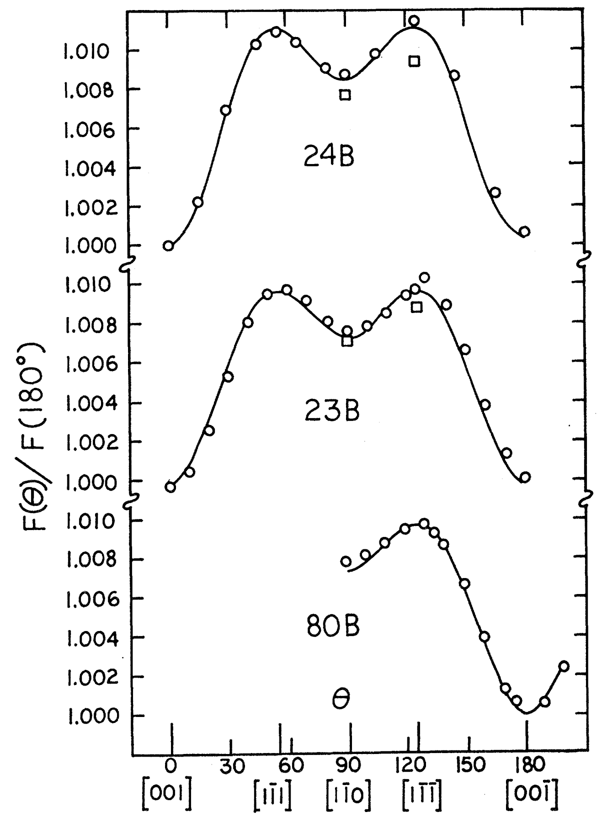

FIG. 1. Angular variation of the frequency ratio for B lying in a (110) plane. The curved lines are obtained from Eq. (5) with the value of C_1 obtained from the data (circles) according to Eq. (7). The squares show the theoretical results taken from Ref. 2.

An analysis of the case where **B** lies in a (110) plane has only been presented here. The other case shown in SB (Fig. 6) was for **B** lying in an approximate (001) plane where no ⟨111⟩ directions are seen. This set of angular data was also rechecked by the analysis presented in this paper. As expected, no significant variations from those reported in SB are seen.

The new experimental data shown in Fig. 1 is in excellent agreement with the warping model used in SB and in good agreement with Zhang's calculations. Thus, it can be concluded that higher-order warping terms in the energy expansion are very small and need not be included to explain the observed SdH frequency anisotropy. Zhang also found that the contribution of the higher-order warping terms was small.

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[5] The values of the theoretical frequencies were taken from Table III of Ref. 2.